## **Compress and Control**



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### A joint effort with...



#### Marc Bellemare and Alvin Chua at AAAI too.



- A meta-algorithm for converting data compression / density estimation algorithms into RL agents.
- E.g. Can make Zip play Pong!

 Builds on earlier compression based classification / clustering work.
[Frank, Chui, Witten, 2000]
[Cilibrasi, Vitanyi, 2005]

#### What is it?

- CnC is a meta-algorithm for policy evalution.
- Converts any compressor / state density model into a policy evaluation algorithm.
- Can be used for heuristic on-policy control.
- Achieves generalization via density estimation; provides an alternative to the usual function approximation route.

#### Not to be confused with...

 Many model-based RL techniques involve learning a model that can imagine the future from the present given the past.



### At a high level

 Determines Q-value by compression similarity of s to previously seen states stratified by return.



### **Problem Setup**

- Assume stationary policy  $\pi$ , *m*-horizon return  $Z_t := \sum_{i=t}^{t+m-1} R_i$ , a stationary MDP environment  $\mu$ , and finite |S|, |A|, |R|.
- Further assume  $\mu + \pi$  gives rise to an ergodic (IR + AP + PR) Markov Chain.
- Goal: Estimate

 $Q^{\pi}(s_t, a_{t+1}) := \mathbb{E}[Z_{t+1} | S_t = s_t, A_{t+1} = a_{t+1}]$ 

#### Intuition

Re-express Q in terms of a time independent distribution:

$$Q^{\pi}(s,a) = \sum_{z \in \mathcal{Z}} z \mathbb{P}(Z = z \mid S = s, A = a)$$

Apply Bayes Rule:

$$Q^{\pi}(s,a) = \sum_{z \in \mathcal{Z}} z \frac{\nu(s \mid z, a) \nu(z \mid a)}{\sum_{z' \in \mathcal{Z}} \nu(s \mid z', a) \nu(z' \mid a)}$$

#### Hold on a minute...

- Conditioning on the future return?!?!?
- We show how this time independent distribution exists and can be learnt online.
- The trick is to construct an augmented, ergdodic HMC whose stationary distribution contains all the information we need.

### **Augmented HMC Construction**

• Can show augmentation preserves ergodicity of underlying ergodic process {  $X_t := (A_t, S_t)$  } given by  $\mu + \pi$ .



### **Stationary Distribution**

- Long term behaviour of the augmented HMC is governed by a unique stationary distribution  $\nu_{\rm w}$
- Then we add on the return Z' i.e.

 $(Z', A'_0, S'_0, R'_0, \dots, A'_m, S'_m, R'_m) \sim \nu$ 

• And can marginalize to get:  $\nu(s, z, a)$ 

#### **Value Estimation**

$$\hat{Q}_t^{\pi}(s,a) := \sum_{z \in \mathcal{Z}} z \, w_t^{z,a}(s)$$

$$w_t^{z,a}(s) := \frac{\rho_{\mathsf{S}}(s \mid s_{0:n-1}^{z,a}) \rho_{\mathsf{Z}}(z \mid z_{1:n}^{a})}{\sum_{z' \in \mathcal{Z}} \rho_{\mathsf{S}}(s \mid s_{0:n-1}^{z',a}) \rho_{\mathsf{Z}}(z' \mid z_{1:n}^{a})}$$

# Algorithm

#### Algorithm 1 CNC POLICY EVALUATION

**Require:** Stationary policy  $\pi$ , environment  $\mathcal{M}$ **Require:** Finite planning horizon  $m \in \mathbb{N}$ **Require:** Coding distributions  $\rho_s$  and  $\rho_z$ 

1: **for** 
$$i = 1$$
 to  $t$  **do**

2: Perform 
$$a_i \sim \pi(\cdot \mid s_{i-1})$$

3: Observe 
$$(s_i, r_i) \sim \mu(\cdot \mid s_{i-1}, a_i)$$

4: **if** 
$$i \ge m$$
 **then**

- 5: Update  $\rho_s$  in bucket  $(z_{i-m+1}, a_{i-m+1})$  with  $s_{i-m}$
- 6: Update  $\rho_{z}$  in bucket  $a_{i-m+1}$  with  $z_{i-m+1}$
- 7: **end if**
- 8: **end for**
- 9: return  $\hat{Q}_t^{\pi}$

#### **Theory overview**

- Consistency of density estimator implies CnC provides consistent value estimates.
- Frequency estimates can be used, and converges stochastically at rate O(n<sup>o.5</sup>)
- CTW can be used for larger problems, idealized version converges stochastically at rate O(n<sup>o.5</sup>)

## **On-policy Control**



Figure 4: Average score over the last 500 episodes for three Atari 2600 games. Error bars indicate one inter-trial one standard error.

### Varying model complexity

A closer look at Pong...



#### Discussion

- Converts the problem of value estimation into one of probabilistic modelling. When is it worthwhile?
- Generalization occurs to the extent it occurs in the density/compression model.
- Seems to work well with essentially bad models. Learning can be quite data efficient.

#### **Future Work**

- Should account for policy drift when doing on-policy control. How?
- Not clear how to do exploration in a principled way for on-policy control.
- Bootstrapping CnC?
- Present work suited for problems where return space is sparse.
- Discretization should be straightfoward, but needs demonstration; needed to run on all Atari games.

#### Questions...

