Reinforcement Learning via AIXI Approximation

Joel Veness[†′] Kee Siong Ng[*] Marcus Hutter[*′] David Silver [‡]

† University of New South Wales
′ National ICT Australia
∗ The Australian National University
‡University College London

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General Reinforcement Learning Problem

Worst case scenario: Environment is unknown. Observations may be noisy. Effects of actions may be stochastic. No explicit notion of state. Perceptual aliasing. Rewards may be sparsely distributed.

Notation:

- Agent interacts with an unknown environment μ by making actions $a \in \mathcal{A}$.
- Environment responds with observations $o \in \mathcal{O}$ and rewards $r \in \mathcal{R}$. For convenience, we sometimes use $x \in \mathcal{O} \times \mathcal{R}$.
- $x_{1:n}$ denotes $x_1, x_2, \ldots x_n$, $x_{<n}$ denotes $x_1, x_2, \ldots x_{n-1}$ and $ax_{1:n}$ denotes $a_1, x_1, a_2, x_2, \ldots a_n, x_n$.

Our work in context

Some approaches to (aspects of) the general reinforcement learning problem:

- Model-free RL with function approximation (e.g. TD)
- POMDP (assume an observation / transition model, maybe learn parameters?)
- Learn some (hopefully compact) state representation, then use MDP solution methods

Our approach:

- Directly approximate Marcus Hutter's AIXI, a Bayesian optimality notion for general reinforcement learning agents.

AIXI: a Bayesian optimality notion

$$a_t^{AIXI} = \arg\max_{a_t} \sum_{x_t} \ldots \max_{a_{t+m}} \sum_{x_{t+m}} \left[\sum_{i=t}^{t+m} r_i \right] \sum_{\rho \in \mathcal{M}} 2^{-K(\rho)} \rho(x_{1:t+m} \,|\, a_{1:t+m}),$$

- Expectimax + (generalised form of) Solomonoff Induction
- Model class $\mathcal{M}$ contains all enumerable chronological semi-measures.
- Kolmogorov Complexity used as an Ockham prior.
- $m := b - t + 1$ is the "remaining search horizon", b is the maximum age of the agent

Caveat: Incomputable. Not an algorithm!

Describing environments, AIXI style

- A *history* h is an element of $(\mathcal{A} \times \mathcal{X})^* \cup (\mathcal{A} \times \mathcal{X})^* \times \mathcal{A}$.
- An *environment* ρ is a sequence of conditional probability functions $\{\rho_0, \rho_1, \rho_2, \dots\}$, where for all $n \in \mathbb{N}$, $\rho_n \colon \mathcal{A}^n \to \textit{Density}\ (\mathcal{X}^n)$ satisfies

$$\forall a_{1:n} \forall x_{<n} : \ \rho_{n-1}(x_{<n} \,|\, a_{<n}) = \sum_{x_n \in \mathcal{X}} \rho_n(x_{1:n} \,|\, a_{1:n}), \rho_0(\epsilon \,|\, \epsilon) = 1.$$

- The ρ-probability of observing x_n in cycle n given history $h = ax_{<n}a_n$ is

$$\rho(x_n \,|\, ax_{<n}a_n) := \frac{\rho(x_{1:n} \,|\, a_{1:n})}{\rho(x_{<n} \,|\, a_{<n})}$$

provided $\rho(x_{<n} \,|\, a_{<n}) > 0$.

Learning a model of the environment

We will be interested in agents that use a *mixture environment model* to learn the true environment μ.

$$\xi(x_{1:n} \,|\, a_{1:n}) := \sum_{\rho \in \mathcal{M}} w_0^{\rho} \rho(x_{1:n} \,|\, a_{1:n})$$

- $\mathcal{M} := \{\rho_1, \rho_2, \dots\}$ is the model class
- w_0^{ρ} is the prior weight for environment ρ.
- Satisfies the definition of an environment model. Therefore, can predict by using:

$$\xi(x_n \,|\, ax_{<n}a_n) = \sum_{\rho \in \mathcal{M}} w_{n-1}^{\rho} \rho(x_n \,|\, ax_{<n}a_n), \;\; w_{n-1}^{\rho} := \frac{w_0^{\rho} \rho(x_{<n} \,|\, a_{<n})}{\sum_{\nu \in \mathcal{M}} w_0^{\nu} \nu(x_{<n} \,|\, a_{<n})}$$

Theoretical Properties

Theorem: Let μ be the true environment. The μ-expected squared difference of μ and ξ is bounded as follows. For all $n \in \mathbb{N}$, for all $a_{1:n}$,

$$\sum_{k=1}^{n} \sum_{x_{1:k}} \mu(x_{<k} \,|\, a_{<k}) \Big(\mu(x_k \,|\, ax_{<k} a_k) - \xi(x_k \,|\, ax_{<k} a_k) \Big)^2 \leq$$

$$\min_{\rho \in \mathcal{M}} \Big\{ - \ln w_0^{\rho} + D_{KL}(\mu(\cdot \,|\, a_{1:n}) \,\|\, \rho(\cdot \,|\, a_{1:n})) \Big\},$$

where $D_{KL}(\cdot \,\|\, \cdot)$ is the KL divergence of two distributions.

Roughly: The predictions made by ξ will converge to those of μ if a model close (w.r.t. KL Divergence) to μ is in $\mathcal{M}$.

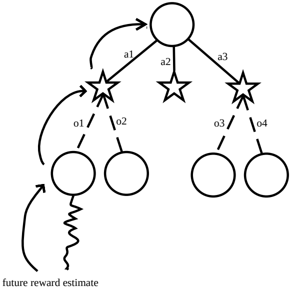
Model Class Approximation

Approximate model class of AIXI with a mixture over *all* action-conditional Prediction Suffix Tree structures of maximum depth D.

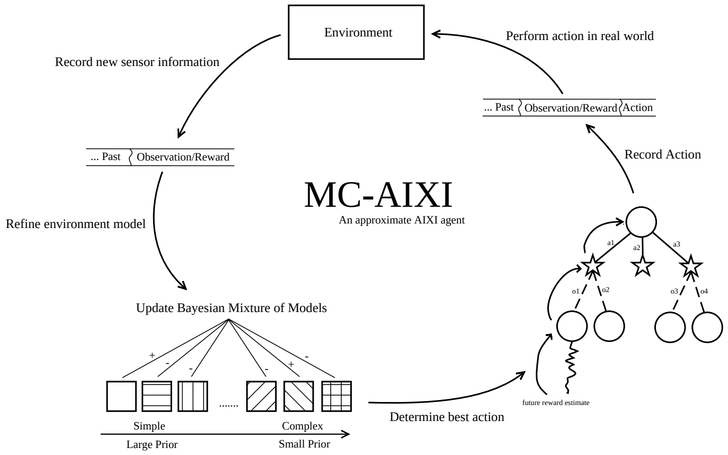
- PSTs are a form of variable order Markov model.
- Context Tree Weighting algorithm can be adapted to compute a mixture of 2^{2^D} environment models in $O(D)$!
- Inductive bias: smaller PST structures favoured.
- PST parameters are learnt using KT estimators. KL-divergence term in previous theorem grows $O(\log n)$.
- Intuitively, efficiency of CTW is due to clever exploitation of shared structure.

ρUCT - MCTS Expectimax Approximation

- Adaptation of UCT for (mixture) environment models
- With sufficient time, converges to the expectimax solution
- "Value of Information" correctly incorporated when instantiated with a mixture environment model.
- Gives Bayesian solution to the exploration/exploitation dilemma.



Agent Architecture



Relationship to AIXI

Given enough thinking time, MC-AIXI(CTW) will choose:

$$a_t = \arg\max_{a_t} \sum_{x_t} \cdots \max_{a_{t+m}} \sum_{x_{t+m}} \left[\sum_{i=t}^{t+m} r_i \right] \sum_{M \in C_D} 2^{-\Gamma_D(M)} \Pr(x_{1:t+m} \,|\, M, a_{1:t+m})$$

In contrast, AIXI chooses:

$$a_t = \arg\max_{a_t} \sum_{x_t} \ldots \max_{a_{t+m}} \sum_{x_{t+m}} \left[\sum_{i=t}^{t+m} r_i \right] \sum_{\rho \in \mathcal{M}} 2^{-K(\rho)} \Pr(x_{1:t+m} \,|\, a_{1:t+m}, \rho)$$

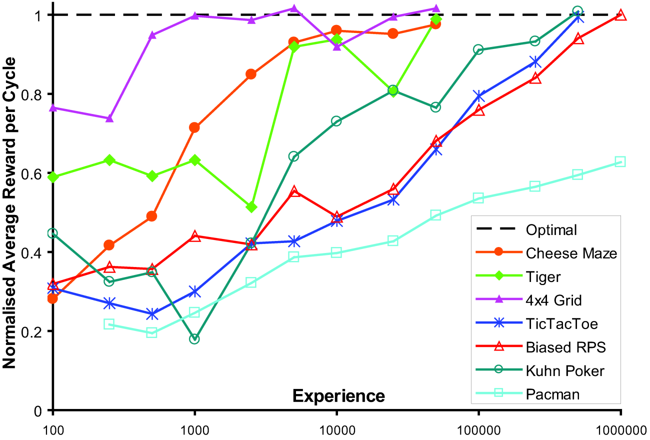
Algorithmic considerations

- Restricted the model class to gain the desirable computational properties of CTW
- Approximated the finite horizon expectimax operation with a MCTS procedure
- $O(Dm \log(|\mathcal{O}||\mathcal{R}|))$ operations needed to generate m observation/reward pairs (for a single simulation)
- $O(tD \log(|\mathcal{O}||\mathcal{R}|))$ space overhead for storing the context tree.
- Anytime search algorithm
- Search can be parallelized
- $O(D \log(|\mathcal{O}||\mathcal{R}|)))$ to update the context tree online

Experimental Setup

- Agent tested on a number of POMDP domains, as well as TicTacToe and Kuhn Poker.
- Agent required to *both* learn *and* plan.
- The context depth and search horizon were made as large as possible subject to computational constraints.
- ϵ-Greedy training, with a decaying ϵ
- Greedy evaluation

Results


Normalised Average Reward per Cycle
1
0.8
0.6
0.4
0.2
0
Experience
100
1000
10000
100000
1000000
Optimal
Cheese Maze
Tiger
4x4 Grid
TicTacToe
Biased RPS
Kuhn Poker
Pacman

Resources required for (near) optimal performance

Domain	Experience	Simulations	Search Time
Cheese Maze	5×10^4	500	0.9s
Tiger	5×10^4	10000	10.8s
4×4 Grid	2.5×10^4	1000	0.7s
TicTacToe	5×10^5	5000	8.4s
Biased RPS	1×10^6	10000	4.8s
Kuhn Poker	5×10^6	3000	1.5s

- Timing statistics collected on an Intel dual quad-core 2.53Ghz Xeon.
- Toy problems solvable in reasonable time on a modern workstation.
- General ability of agent will scale with better hardware.

Limitations and Future Work

- PSTs inadequate to represent many simple models compactly. For example, it would be unrealistic to think that our current AIXI approximation could cope with real-world image or audio data.
- Exploration/exploitation needs more attention. Can something principled *and* efficient be done for general Bayesian agents using large model classes?

Future Work

- Uniform random rollout policy used in ρUCT. A learnt policy should perform much better.
- All prediction was done at the bit level. Fine for a first attempt, but no need to work at such a low level.
- Mixture environment model definition can be extended to continuous model classes.
- Incorporate more (action-conditional) Bayesian machinery.
- Richer notions of context.

For the curious...

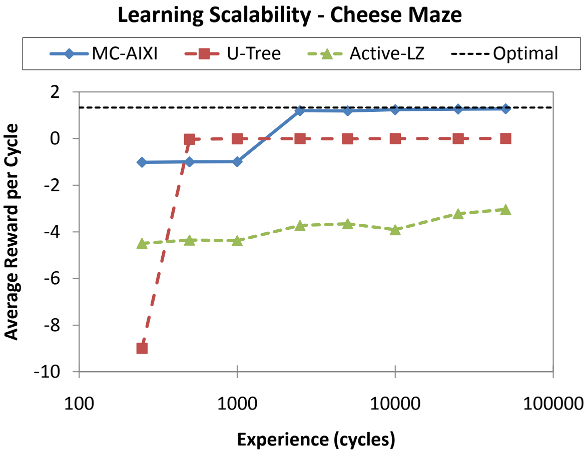
- For more information, see:

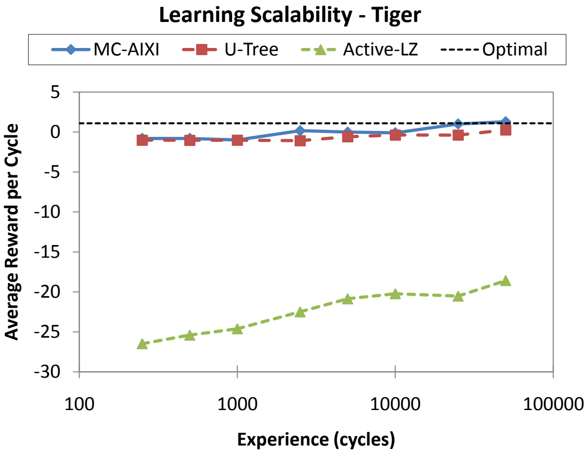
  A Monte-Carlo AIXI Approximation,
  J. Veness, K.S. Ng, M. Hutter, W. Uther, D. Silver
  http://jveness.info/publications/default.html

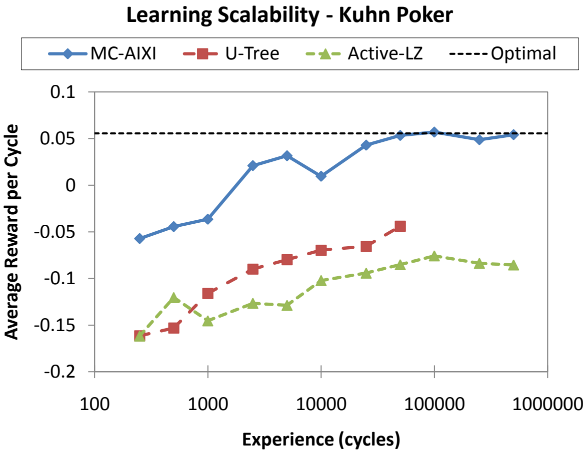
  Highlights: a direct comparison to U-Tree / Active-LZ, improved model class approximation (FAC-CTW) and more relaxed presentation.

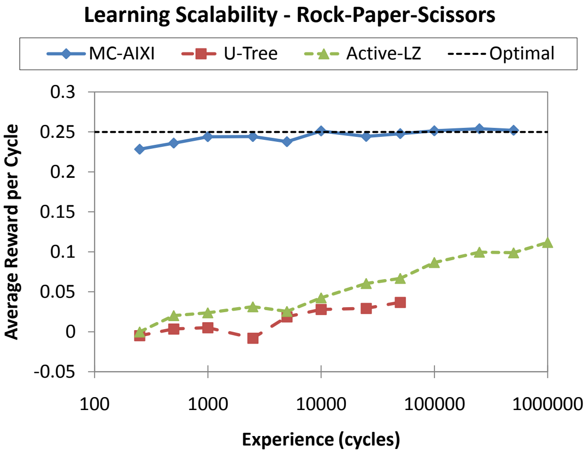
- Video of the latest version playing Pacman
  http://www.youtube.com/watch?v=yfsMHtmGDKE

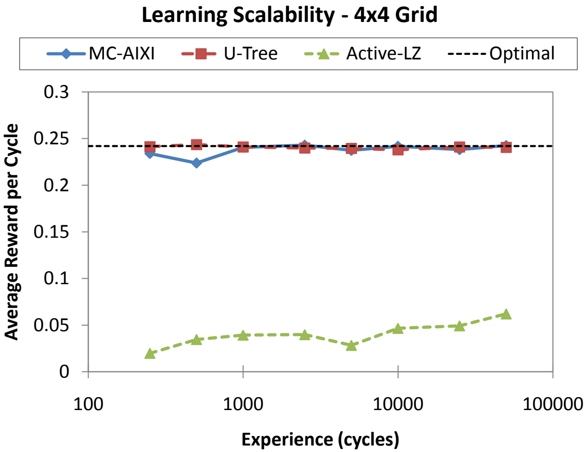
- Source code at: http://jveness.info/software/default.html

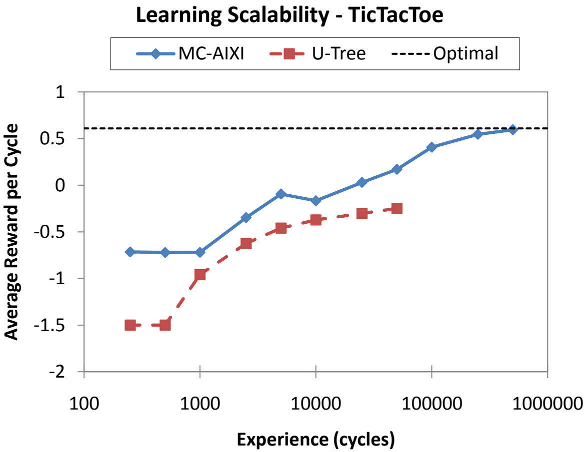

Learning Scalability - Cheese Maze
MC-AIXI
U-Tree
Active-LZ
Optimal
Average Reward per Cycle
2
0
-2
-4
-6
-8
-10
100
1000
10000
100000
Experience (cycles)


Learning Scalability - Tiger
MC-AIXI
U-Tree
Active-LZ
Optimal
5
0
-5
-10
-15
-20
-25
-30
100
1000
10000
100000
Average Reward per Cycle
Experience (cycles)


Learning Scalability - Kuhn Poker
MC-AIXI
U-Tree
Active-LZ
Optimal
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
Average Reward per Cycle
100
1000
10000
100000
1000000
Experience (cycles)


Learning Scalability - Rock-Paper-Scissors
MC-AIXI
U-Tree
Active-LZ
Optimal
Average Reward per Cycle
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
100
1000
10000
100000
1000000
Experience (cycles)


Learning Scalability - 4x4 Grid
MC-AIXI
U-Tree
Active-LZ
Optimal
0.3
0.25
0.2
0.15
0.1
0.05
0
Average Reward per Cycle
100
1000
10000
100000
Experience (cycles)


Learning Scalability - TicTacToe
MC-AIXI
U-Tree
Optimal
1
0.5
0
-0.5
-1
-1.5
-2
Average Reward per Cycle
100
1000
10000
100000
1000000
Experience (cycles)

Questions?

- Thanks for coming, I hope you enjoyed my talk.