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Proton Spin in the Instanton Background

Marcus Hutter[1]

Sektion Physik der Universität München
Theoretische Physik
Theresienstr. 37 80333 München

Abstract

The proton form factors are reduced to vacuum correlators of 4 quark fields by assuming independent constituent quarks. The axial singlet quark and gluonic form factors are calculated in the instanton liquid model. A discussion of gauge(in)dependence is given.

[1] E–Mail:hutter@hep.physik.uni–muenchen.de

Contents

1 Introduction

A variety of predictions concerning chiral symmetry breaking can be made within the instanton liquid model. Although the 't Hooft interaction [24] explicitly breaks the $U(1)$ axial symmetry, instanton models are up to now not too successful in describing quantitatively the axial singlet channel. The most interesting quantities are the η' mass and the spin of the proton.

Sections 2, 3 and 4 are an introduction to the proton spin problem. In section 5 the proton form factors are reduced to vacuum correlators of 4 quark fields by assuming independent constituent quarks. The axial singlet quark and gluonic form factors are calculated in section 6, 7 and 8 by using the propagator and 4 point functions of the instanton liquid model. Gauge(in)dependence is examined. A discussion of the results and a comparison with [25] is given in section 9.

2 Measurement of the Axial Form Factors

The forward matrix elements of the axial currents

$$s_\mu \Delta\psi = \langle ps|\bar{\psi}\gamma_\mu\gamma_5\psi|ps\rangle \quad , \quad \psi = u, d, s$$

can be interpreted as the quark spin content of the proton, in a sense defined more accurately in the following sections. Three independent linear combinations of $\Delta\psi$ have been measured, thus allowing to extract their individual values.

From the neutron β-decay, using isospin invariance, one gets [7, 10]

$$a_3 = g_A = \Delta u - \Delta d = F + D = 1.254 \pm 0.06 \quad .$$

From the octet hyperon β-decay, using $SU(3)_F$ symmetry, one gets [9, 10]

$$\sqrt{3}a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D = 0.688 \pm 0.0035 \quad .$$

From the spin dependent structure function g_1^p of the proton, which has been measured by EMC [11] and SMC [12], one can extract

$$\Gamma_p = \int_0^1 g_1^p(x)dx = \frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s + O(\alpha_s) = 0.142 \pm 0.014 \tag{1}$$

where we have given the world average value.

Of special interest is the quarkspin sum, which can be extracted from the values given above,

$$\Delta\Sigma^{GI} = \sqrt{\frac{3}{2}}a_0 = \Delta u + \Delta d + \Delta s = 0.27 \pm 0.13 \tag{2}$$

where the $O(\alpha_s)$ corrections have been included. It deviates significantly from the naive quark model value $\Delta\Sigma_{qm} = 1$. This deviation is the origin of the so called spin problem. Further the large polarization of strange quarks in the proton

$$\Delta s = -0.1 \pm 0.05$$

is counter intuitive because this indicates a large strange quark content of the proton.

Much more could be said about proton spin phenomenology and the experiments. For an introduction and further references see [14, 15, 16, 11]. We will now give a more thorough definition and interpretation of $\Delta\Sigma^{GI}$ and other quantities, which we want to calculate within the instanton model.

3 Axial Singlet Currents & Anomaly

It is well known that products of operators at the same spacetime point are very singular objects. In order to make the expressions well defined one has to regularize and renormalize the operator products. An anomaly appears, if this procedure breaks a symmetry of the theory. The most important ones are the breakdown of the scale invariance and the breakdown of the axial symmetry [17]. In the following we are interested in the axial anomaly [18]. The operator product which has to be regularized is the axial singlet current,

$$J_{\mu 5}(x) = \sum_{q\in\{u,d,s,\ldots\}} \bar{q}(x)\gamma_\mu\gamma_5 q(x) \tag{3}$$

which seems to be local, gauge invariant and conserved[2]. Unfortunately after regularization one of the three properties is unavoidably lost. Therefore we can define two different local currents, a conserved (c) one and a gauge invariant (GI) one. The third GI, conserved and non-local current is discussed in [19] in connection with the $U(1)$ problem. We will suppress the summation over quark flavors and write ψ for the quark field operator:

$$\begin{aligned} J^{GI}_{\mu 5}(x) &= \lim_{\varepsilon\to 0} \bar\psi(x+\varepsilon)\gamma_\mu\gamma_5 P\exp\left(i\int_x^{x+\varepsilon} dz\cdot A(z)\right)\psi(x) \\ J^{c}_{\mu 5}(x) &= \lim_{\varepsilon\to 0} \bar\psi(x+\varepsilon)\gamma_\mu\gamma_5\psi(x) \end{aligned} \tag{4}$$

The difference between the two currents is described by the anomaly current K_μ:

$$\begin{aligned} K_\mu(x) &= \frac{N_f\alpha_s}{2\pi}\varepsilon_{\mu\nu\rho\sigma}\mathrm{tr}_c A^\nu(G^{\rho\sigma}-\frac{2}{3}A^\rho A^\sigma) \quad , \quad J^{GI}_{\mu 5} = J^c_{\mu 5} + K_\mu \\ \partial^\mu K_\mu(x) &= \frac{N_f\alpha_s}{2\pi}\mathrm{tr}_c G\tilde G(x) = a(x) \\ \partial^\mu J^c_{\mu 5}(x) &= 2mJ_5(x) \qquad , \quad J_5 = i\bar\psi\gamma_5\psi \quad . \end{aligned} \tag{5}$$

m is the current quark mass and N_f is the number of quark flavors. Note, that the splitting of $J_{\mu 5}$ in a conserved and an anomaly part is gauge dependent. There are attempts to define both uniquely on physical grounds [15]. The intention is to define $J^c_{\mu 5}$ as the naive parton model spin and K_μ as some gluonic contribution. The proton matrix elements of the various currents can be expressed in terms of real form factors $G_i, K_i, AandJ$:

$$\begin{aligned} \langle p's'|J^{GI}_{\mu 5}(0)|ps\rangle &= \bar u_{s'}(p')\left[\gamma_\mu\gamma_5 G_1^{GI}(q^2) - q_\mu\gamma_5 G_2^{GI}(q^2)\right]u_s(p) \\ \langle p's'|J^{c}_{\mu 5}(0)|ps\rangle &= \bar u_{s'}(p')\left[\gamma_\mu\gamma_5 G_1^{c}(q^2) - q_\mu\gamma_5 G_2^{c}(q^2)\right]u_s(p) \\ \langle p's'|K_\mu(0)|ps\rangle &= \bar u_{s'}(p')\left[\gamma_\mu\gamma_5 K_1(q^2) - q_\mu\gamma_5 K_2(q^2)\right]u_s(p) \\ \langle p's'|a(0)|ps\rangle &= 2MiA(q^2)\bar u_{s'}(p')\gamma_5 u_s(p) \\ \langle p's'|J_5(0)|ps\rangle &= iJ(q^2)\bar u_{s'}(p')\gamma_5 u_s(p) \end{aligned} \tag{6}$$

M is the proton mass and $q = p' - p$. From (5) one can derive the following relations between the form factors:

$$\begin{gathered} G_1^{GI} = G_1^c + K_1 \quad , \quad G_2^{GI} = G_2^c + K_2 \\ G_1^c - \frac{q^2}{2M}G_2^c = \frac{m}{M}J \quad , \quad K_1 - \frac{q^2}{2M}K_2 = A \\ G_1^{GI} - \frac{q^2}{2M}G_2^{GI} = \frac{m}{M}J + A \quad , \end{gathered} \tag{7}$$

where all form factors are evaluated at q. The last equation relates only GI quantities. In the next section we show that the form factor G_1^{GI} at zero momentum transfer can be connected with the proton spin.

[2] We will use the term 'conserved' even for $m_q \neq 0$. Sometimes this current is called the symmetric current in the literature.

4 The Proton Spin and its Interpretation

The stress tensor $T_{\mu\nu}$ is conserved ($\partial_\mu T^{\mu\nu} = 0$) symmetric and GI and can be constructed from the Noether theorem. The angular momentum density tensor $M^{\mu\nu\rho}$ associated with Lorentz transformations can be expressed in terms of $T_{\mu\nu}$:

$$M^{\mu\nu\rho} = x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu} \tag{8}$$

M can be decomposed in spin and orbital contribution of quarks and gluons [10]:

$$\begin{aligned} M^{\mu\nu\rho} &= M^{\mu\nu\rho}_{q,orb} + M^{\mu\nu\rho}_{q,spin} + M^{\mu\nu\rho}_{g,orb} + M^{\mu\nu\rho}_{g,spin} - \frac{1}{4}G^2(x^\nu g^{\mu\rho} - x^\rho g^{\mu\nu}) + \partial(\cdots) \\ M^{\mu\nu\rho}_{q,orb} &= \frac{1}{2} i\bar{\psi}\gamma^\mu(x^\nu\partial^\rho - x^\rho\partial^\nu)\psi \quad , \quad M^{\mu\nu\rho}_{q,spin} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi = \frac{1}{2}J^{GI}_{\sigma 5} \\ M^{\mu\nu\rho}_{g,orb} &= -G^{\mu\sigma}(x^\nu\partial^\rho - x^\rho\partial^\nu)A_\sigma \quad , \quad M^{\mu\nu\rho}_{g,spin} = G^{\mu\rho}A^\nu - G^{\mu\nu}A^\rho \end{aligned} \tag{9}$$

The last two terms in $M^{\mu\nu\rho}$ do not contribute to the angular momentum operator

$$J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x\ M^{0jk}(x) \quad . \tag{10}$$

Taking the matrix element of J_z in a proton state, where the proton is aligned in z-direction and at rest we get the spin of the proton

$$\Delta J = \frac{1}{\mathcal{N}}\langle ps|J_z|ps\rangle = \frac{1}{2}\varepsilon^{3jk}\langle ps|M^{0jk}(0)|ps\rangle \quad , \quad \mathcal{N} = \langle p,s|p,s\rangle = \delta^{\,3}(0) \tag{11}$$

The total spin of the proton is with no doubt 1/2 and we get the sum rule

$$\Delta J = \Delta L_q + \frac{1}{2}\Delta\Sigma^{GI} + \Delta L_g + \Delta g = \frac{1}{2} \tag{12}$$

where $(\Delta L_q, \frac{1}{2}\Delta\Sigma^{GI}, \Delta L_g, \Delta g)$ are the (quark-orbital, quark-spin, gluon-orbital, gluon-spin) contribution to the proton spin, defined as matrix elements of the various parts of M given above. Therefore The GI axial current measures the quark spin contribution to the proton spin. The space integral in 10 can be cancelt with the state normalization and we get in covariant notation:

$$s_\mu\Delta\Sigma^{GI} = \langle ps|J^{GI}_{\mu 5}(0)|ps\rangle \quad \Longrightarrow \quad \Delta\Sigma^{GI} = G^{GI}_1(0) \tag{13}$$

In the naive quark model the proton consists of three quarks at rest. There is no orbital and no gluonic contribution to the proton spin. This leads to the Ellis-Jaffe sum rule $\Delta J = \frac{1}{2}\Delta\Sigma^{GI} = 1/2$. In the real world the identification of $\Delta\Sigma^{GI}$ with the proton spin is not correct, because $J^{GI}_{\mu 5}$ measures the spin of the (nearly massless) current quarks whereas the proton consists of three massive (≈ 300 MeV) constituent quarks. Further in a model of non-interacting constituent quarks the axial current which measures the constituent quark spin should be anomaly free because the anomaly is due to the interaction with gluons. Therefore the conserved current $J^c_{\mu 5}$ might be identified with the constituent quark spin operator.

$$s_\mu\Delta\Sigma^c = \langle ps|J^c_{\mu 5}(0)|ps\rangle \quad \Longrightarrow \quad \Delta\Sigma^c = G^c_1(0) \stackrel{?}{=} 1 \quad . \tag{14}$$

From (7) we get

$$\Delta\Sigma^{GI} = \Delta\Sigma^c + K_1(0) \tag{15}$$

which can now be interpreted in the following way: The spin of the constituent quarks $\Delta\Sigma^c$ are formed by the spin of the current quarks $\Delta\Sigma^{GI}$ and a rest $-K_1(0)$, which contains orbital and gluonic contributions. The origin of these contributions is *not* the motion and interaction of the constituent quarks inside the proton, because the constituent quarks are noninteracting and at rest in the naive quark model, but due to the formation of massive quarks from massless quarks. Therefore (15) may be discussed for an individual "constituent" quark. Further the gluonic configurations which are responsible for the generation of the quarkmass also determine the value of $K_1(0)$.

E.g. in a BAG model a massive quark is formed by confining a massless quark to a sphere. The spin of the massive constituent quark is the sum of the spin ($\frac{1}{2}\Delta\Sigma^{GI}$) and the oribtal $\frac{1}{2}(1-\Delta\Sigma^{GI})$ contribution of the current quark. The BAG, which might be formed by nonperturbative gluonic configurations, is responsible for the mass generation and indirectly for the orbital contribution. From analytical and numerical calculations we know, that in the BAG model the constituent spin is splitted into 70% spin and 30% orbital contribution when starting with massless quarks.

Whereas (15) is rigorously true, the interpretation of $\frac{1}{2}\Delta\Sigma^c$ as the spin of a constituent quark and its value $\frac{1}{2}$ is questionable. One reason is, that an axial current which describes massive constituent quarks is by no means conserved in contradiction to $J^c_{\mu 5}$.

There exists another relation between $\Delta\Sigma^{GI}$ and the form factor A at zero momentum transfer. Before deriving this relation we have to give a short discussion about the order of limits and massless poles. The following limits are taken: The spacetime volume goes to infinity ($V_4 \to \infty$), because the universe is actually very large, the current quark masses go to zero ($m \to 0$), because the up and down masses are very small and $q \to 0$, because we are interested in the forward matrix elements. In principle the results can depend on the order of the limits and therefore they have to be choosen consistent with the physical situation. This means, that if in the real world e.g. $q \ll m$ we first have to take $q \to 0$ and then $m \to 0$. Actually we are interested in the forward matrix element ($q \equiv 0$) and $m \neq 0$ in the real world and the order of limits just stated applies. Through the cluster theorem connected correlators in coordinate space have to decay to zero when the separation of two arguments tends to infinity. Therefore there are no $\delta(q)$-peaks in momentum space and the order of limits $q \to 0$ and $V_4 \to \infty$ can be taken at will. Because $m^4 V_4 \gg 1$ we have to take first $V_4 \to \infty$ and then $m \to 0$. In statistical physics this is a well known fact, that a spontaneous breakdown of a symmetry only occurs, when there is a small explicit symmetry breaking term and the system volume tends to infinity. In the final end one may remove the symmetry breaking term. In QCD chiral symmetry is spontaneously broken (SBCS) and the small current quark mass is the explicit breaking of the chiral symmetry. Therefore it is mandatory first to take $V_4 \to \infty$ and then $m \to 0$ [21]. Therefore we can use the following order of limits

$$\lim_{m\to 0}\{\lim_{q\to 0}[\lim_{V_4\to\infty}(\ldots)]\}. \tag{16}$$

This justifies the usage of the infinite volume formulation from the very beginning.

In real QCD there are no massless particles ($m_\pi \neq 0$). Therefore GI form factors have no massless poles especially

$$q^2 G_2^{GI}(q^2) \xrightarrow{q \to 0} 0 \tag{17}$$

From (7), (13) and (17) we get

$$\Delta\Sigma^{GI} = \frac{m}{M} J(0) + A(0) \tag{18}$$

This relation is true wether there are Goldstone bosons in the axial singlet channel or not. Experimentally we know that the lightest particle in this channel is the η' with a mass of 958 MeV much too large to be a Goldstone boson. Therefore $J(0)$ remains finite in the chiral limit and we obtain

$$\Delta\Sigma^{GI} = A(0) \quad \text{for} \quad m \to 0 \tag{19}$$

Assuming the non-existence of the axial singlet Goldstone boson from the very beginning the order of limits is of no importance in deriving (19). Note, that (19) is only true, if we take $m = m_u = m_d = m_s$, although all three masses tend to zero. Otherwise additional nonsinglet currents on the r.h.s. of (18) would survive the chiral limit [8].

Combining (7), (17) and (15) we can conclude that

$$2M\Delta\Sigma^c = q^2 G_2^c(q^2)_{|q^2=0} = -q^2 K_2(q^2)_{|q^2=0}$$

$\Delta\Sigma^c$ is given by the pole residuum of G_2^c. Because G_2^{GI} has no massless pole $\Delta\Sigma^c$ is also given by the pole of $-K_2$. These massless poles are called ghost poles and they may truly appear, even if there are no physical massless particles, because G_2^c and K_2 are gauge dependent objects. Note that all other form factors defined in (7) are GI and therefore free of massless poles.

Table 4 summarizes the values for the form factors at zero momentum transfer for the following three cases:

- the naive quark model of non-interacting constituent quarks of mass $m = M/N_f$,
- chiral QCD and the identification of $\Delta\Sigma^c$ with the naive spin value 1,
- the instanton liquid model.

In the following sections we will calculate some of the form factors for a single constituent quark in the instanton liquid model.

5 Reduction of the Proton Form Factors to Vacuum Correlators

In this section we will calculate some of the form factors defined above in the instanton liquid model. To apply the methods developed in [2] we relate the form factors to vacuum correlation functions

$$\langle p's'|B(0)|ps\rangle = \tag{20}$$

	$\Delta\Sigma^c = \frac{q^2}{2M} G_2^c + \frac{m}{M} J$	$K_1 = \frac{q^2}{2M} K_2 + A$	$\Delta\Sigma^{GI} - \frac{q^2}{2M} G_2^{GI} = \Delta\Sigma^c + K_1 = \frac{m}{M} J + A$
$N_f m = M$	1 = 0 + 1	0 = 0 + 0	1 - 0 = 1 + 0 = 1 + 0
$m = 0$	1 = 1 + 0	A-1= -1 + A	A - 0 = 1 +A-1= 0 + A
Instanton	? = ? + 0	0 = 1 +(-1)	1 - 0 = ? + 0 = 0 +(-1)

Table 1: *The proton form factors at zero momentum transfer $q^2 = 0$ in the naive constituent quark model ($N_f m = M$), in chiral QCD (m=0) and in the instanton-liquid model (Instanton). Experimentally A is 0.27.*

$$= -\frac{1}{Z_\eta} \bar{u}_{s'}(p') \left[\int d^4x\, d^4z\; e^{ip'x - ipz} (i\partial\!\!\!/_x - M)(-i\partial\!\!\!/_z - M) \langle 0 | \mathcal{T} \eta(x) B(0) \bar{\eta}(z) | 0 \rangle \right] u_s(p) \quad .$$

M is the proton mass and $B(0)$ is an arbitrary local operator. $\eta(x)$ is a local operator with the quantum numbers of a proton e.g. a product of three quark fields in an appropriate spin and flavor combination [20]. Assuming that $\eta(x)$ tends to a free proton field operator for infinit times the proton states can be reduced and (20) is just an LSZ reduction formula for composite fields. For our purpose the following form is more suitable

$$\begin{aligned} \langle p's' | B(0) | ps \rangle &= Z_\eta \bar{u}_{s'}(p') [\lim_{p^2, p'^2 \to M^2} S^{-1}(p') T_B(p', p) S^{-1}(p)] u_s(p) \\ T_B(p', p) &= \int d^4x\, d^4z\; e^{ip'x - ipz} \langle 0 | \mathcal{T} \eta(x) B(0) \bar{\eta}(z) | 0 \rangle \\ S(p) &= \int d^4x\; e^{ipx} \langle 0 | \mathcal{T} \eta(x) \bar{\eta}(0) | 0 \rangle = \frac{i Z_\eta}{p\!\!\!/ - M} + continuum \\ Z_\eta^{1/2} u_s(p) &= \langle 0 | \eta(0) | ps \rangle \end{aligned} \tag{21}$$

The advantage of this form is, that the explicit knowledge of the mass M is not needed. In Euclidian calculations like lattice-, instanton- and OPE-calculations it is always difficult to extract pole masses.

This form can also be interpreted as a spectral representation of the 3 point function. Inserting two complete sets of states into the 3 point function and taking the limit $p^2 = p'^2 \to M^2$ to select the proton state one can directly attain (21).

If e.g. $B(0)$ is a quark current, the 3 point function is a product of 8 quark fields, which is too complicated to be evaluated in a multi-instanton background. Let us assume that the proton consists of three nearly independent quarks. Then the main nonperturbative properties of the proton come from the formation of constituent quarks out of current quarks. The forces which confine the constituent quarks in the proton are assumed to modify the properties of the proton only in a minor way, except that the proton is then stable. This assumption is justified by the success of the constituent quark model. The form factors of the proton are therefore the sum of the form factors of the constituent quarks. η has to be replaced by a single quark field ψ of flavor *up* or *down* and M must be replaced by the constituent quark mass. In this case it is even more important to use (21) because one does not expect a definit pole mass for the quark propagator. Looking at the quark propagator in the instanton liquid model we see, that the $p\!\!\!/$ term remains unrenormalized and therefore $Z_\psi = 1$. For a constant constituent mass this argument would be rigorously true. For a running mass it is plausible that Z_ψ is still approximately one. This fact is true in all models of chiral symmetry breaking I know. A conservative estimate is

$$0.7 \leq Z_\psi \leq 1 \tag{22}$$

In the following we will set $Z_\psi = 1$ remembering that this not an exact statement. The results for all form factors have to be multiplied with Z_ψ.

6 The Axial Form Factors $G_{1/2}^{GI}(q)$

The form factor of the current $j_\Gamma = \bar\psi\Gamma\psi$ of a constituent quark can be reduced with the help of (21) to a 4 point function

$$tr_{CD}[T_{j_\Gamma}(p',p)\Gamma'] = \int d^4x\, d^4z\ e^{ip'x-ipz} tr_{CD}[\langle 0|\mathcal{T}\psi(x)\bar\psi(0)\Gamma\psi(0)\bar\psi(z)|0\rangle\Gamma'] = \tag{23}$$

$$= \int d^4q\ \Pi_{\Gamma\Gamma'}(q-p, q-p', p, p')$$

The polarisation functions $\Pi_{\Gamma\Gamma'}$ are calculated are defined and calculated in the instanton liquid model in [2] and other works. For $\Gamma = \gamma_\mu\gamma_5$ the connected part of the 4 point function is suppressed by $O(n_R^{1/2})$. In leading order in the instanton density only the disconnected part contributes and we get

$$T_{j_{\mu 5}^{GI}}(p',p) = S(p')\gamma_\mu\gamma_5 S(p) \tag{24}$$

Inserting (24) in (21) and comparison with (6) leads to

$$\langle p's'|J_{\mu 5}^{GI}(0)|ps\rangle = \bar u_{s'}(p')\gamma_\mu\gamma_5 u_s(p) \tag{25}$$

$$G_1^{GI}(q^2) = 1 \quad , \quad G_2^{GI}(q^2) = 0$$

Note, that $\Pi_{\Gamma\Gamma'}$ was calculated in singular gauge, but the connected part is suppressed in any gauge and the disconnected part only depends on the propagators, which cancel out anyway. The form factors $G_{1/2}^{GI}(q)$ are indeed gauge invariant. The result coincides with a model of free massive quarks. Further we see that the current is not conserved. Conservation depends $q^2G_2 = MG_1$, which is clearly not satisfied by (25). In the one instanton approximation one can work from the very beginning with the effective 't Hooft vertex [24] which explicitly breaks the $U(1)$ symmetry and therefore contains the anomaly.

The result for the GI form factors (25), although not consistent with the experimental value, is *up to now* at least theoretical consistent.

7 The Anomaly Form Factor $A(q)$ *

We will now calculate the anomaly form factor A. Using again the reduction formula with insertion of the anomaly current $B(0) = a(0)$ we have to calculate the 3 point function $T_a(p, s)$. In the instanton model the field operator $a(0)$ is replaced by a classical field $a_A(0)$ where $A = \sum_I A_I$ is a multi instanton configuration inserted in a. In a given background A the correlator can be written in the form

$$\langle 0|\mathcal{T}\psi(x)a(0)\bar{\psi}(z)|0\rangle_A = a_A(0)\langle 0|\mathcal{T}\psi(x)\bar{\psi}(z)|0\rangle_A = a_A(0)S_A(x, z) \tag{26}$$

where $S_A(x, z)$ ist the quark propagator in the multi instanton background A. The r.h.s. has now to be averaged over the collective coordinates γ_I of all instantons. Without the factor $a_A(0)$ this is just the averaged quark propagator calculated in [2]. $a_A(y)$ is $2N_f$ times the topological charge density at spacetime point y. In the vicinity of an instanton of charge $Q_I = \pm 1$ the charge density has a positive/negative bump and is small elsewhere. Therefore $a_A(y)$ is only nonzero when there is at least one instanton near y. Let us fix exactly one instanton in the vicinity of $y = 0$. The orientation and charge of the remaining instantons can be averaged independently, but when averaging the locations z_I the domain near y has to be avoided. The next step is to assume 2 instantons near y and so on. The relative error we make by neglecting these further contributions and by forgetting about the restriction on z_I are both of $O(n_R)$. In leading order in the instanton density we can therefore fix one instanton near $y = 0$ and take only this contribution to $a_A(0)$ into account. The remaining instantons can be averaged as in the pure propagator case and the diagrams which have to be summed and averaged are the same except for the fixing of one instanton I. The propagator consists of a chain of instanton scatterings A_J $(J = 1 \ldots N)$. Repeated scattering at this vertex is allowed. There are two cases: The first case is that all instantons left to all occurrences of instanton I are different to all instantons right to all occurences of instanton I. In leading order in $1/N_c$ all instantons in the middle section from the first up to the last occurence of A_I are different to the exterior instantons. The instantons on the left and on the right can be averaged independently leading to averaged multi-instanton propagators. Averaging the middle section, but fixing I leads to the effective vertex M_I. The free part of the correlator in momentum space is therefore

$$\begin{aligned} T_a^{free}(p, s) &= \langle 2N_f Q_I(z_I) \; \xleftarrow[p]{} M_I \xleftarrow[s]{} \rangle_I = \\ &= -2iN_f \hat{Q}(p - s)\sqrt{M_p M_s}S(p)\gamma_5 S(s) \\ Q_I(z_I) &= \frac{1}{2N_f} a_{A_I}(0) = \pm\frac{6}{\pi^2}\left(\frac{\rho}{z_I^2 + \rho^2}\right)^4 \end{aligned} \tag{27}$$

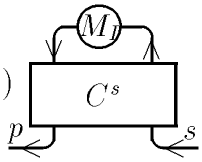
$Q_I(z_I)$ is the charge density of one instanton of charge $Q_I = \pm 1$ and $\hat{Q}(q) = \frac{1}{2}(q\rho)^2 K_2(q\rho)$ its fourier transform[3] for $Q_I = +1$. For $p^2 = s^2 = M^2$ the term $\sqrt{M_p M_s}$ is just the onshell

[3] I apologize for the overload of the symbol K: $K_2(q\rho)$ is a modified Bessel function, $K_\mu(x)$ is the anomaly current and $K_{1/2}(q)$ its form factors.

mass M. Inserting (27) into (21) and comparision with (6) we get for the free part of the anomaly form factor:

$$A^{free}(q) = -N_f\hat{Q}(q) \quad , \quad A^{free}(0) = -N_f \tag{28}$$

The second case is, that there are common instantons to the left and to the right of instanton I. The connected part of the correlator and the form factor are

$$T^{conn}_a(p,s) = \Big\langle 2N_f Q_I(z_I)$$


$$\Big\rangle_I = \tag{29}$$

$$= -4(N_f-1)i\hat{Q}(p-s)C^s_5(p-s)F_5(p-s)\sqrt{M_pM_s}S(p)\gamma_5 S(s)$$

$$A^{conn}(q) = -2(N_f-1)\hat{Q}(q)C^s_5(q)F_5(q) \quad , \quad A^{conn}(0) = N_f - 1 \tag{30}$$

For one flavor the connected part is zero as it should. For two flavors the result can easily derived by using the formulas of [2]. The total anomaly form factor for zero momentum transfer

$$A(0) = A^{free}(0) + A^{conn}(0) = -1 \tag{31}$$

is independent of the number of flavors! This result is welcomed due to the following argument: The form factors of the axial singlet currents $j_{\mu 5}$ should not depend on any quark flavor which is not involved in the particle state. One expects that they are independent of N_f. Due to (5) matrix elements of $a(x)$ must then be independent of N_f too. But this is not obvious because $a(x)$ is explicitly proportional to N_f and the gluonic field is not flavor sensitive. The calculation given above shows how the quark interaction cancels the free part, which is proportional to N_f, so that the total form factor is independent of N_f at least at zero momentum transfer.

8 The Gluonic Form Factors $K^{GI}_{1/2}(q)$

Now we come to the calculation of $K_{1/2}(0)$. The previous calculation can be copied with minor changes. $a(0)$ has to be replaced by $K_\mu(0)$. This in turn induces the replacement

$$2Q(z_I) \rightsquigarrow G_\mu(z_I) := \frac{1}{N_f}K^\mu_{A_I}(0) \quad , \quad 2\hat{Q}(q) \rightsquigarrow \hat{G}_\mu(q) \tag{32}$$

$G_\mu(z_I)$ is $K_\mu(0)$ where the gauge field is an instanton centered at z_I of charge $Q_I = +1$ and $\hat{G}_\mu(q)$ is its fourier transform. In regular gauge we get

$$\begin{aligned} G^{reg}_\mu(z) &= \frac{1}{N_f}K^\mu_{A^{reg}_I}(0) = -\frac{z_\mu(z^2+3\rho^2)}{\pi^2(z^2+\rho^2)^3} \\ \hat{G}^{reg}_\mu(q) &= -iq_\mu\rho^2K_2(q\rho) \xrightarrow{q\to 0} -2iq_\mu/q^2 \end{aligned} \tag{33}$$

With this replacement in (27) and (29) and comparison with (6) $K_{1/2}(0)$ can be extracted:

$$K_1^{reg}(q)=0 \quad , \quad \lim_{q^2\to 0}\frac{q^2}{2M}K_2^{reg}(q)=1 \tag{34}$$

In singular gauge we get

$$G_\mu^{sing}(z)=G_\mu^{reg}(z)+\frac{z_\mu}{\pi^2 z^4} \quad , \quad \hat{G}_\mu^{sing}(q)=\hat{G}_\mu^{reg}(q)+2iq_\mu/q^2 \stackrel{q\to 0}{\longrightarrow} 0 \tag{35}$$

$$K_1^{sing}(q)=0 \quad , \quad \lim_{q^2\to 0}\frac{q^2}{2M}K_2^{sing}(q)=0$$

An apparent observation is, that the anomaly form factor $K_2(q)$ is gauge dependent and recieves a massless pole in regular gauge. The reason for this is the gauge dependence of the anomaly current K_μ itself. One can show that the forward matrix elements $K_{1/2}(0)$ are GI for small gauge transformations. A gauge transformation is called small, when it can be smoothly deformed into the unit transformation. On the other hand the gauge transformation, which transforms an instanton from regular gauge to one in singular gauge is large, because the regular solution can not be smoothly deformed into a singular one due to the singularity.

The next striking observation is that the relation

$$K_1(q)-\frac{q}{2M}K_2(q)=A(q) \tag{36}$$

is violated in singular gauge as can be seen from (35)

$$K_1^{sing}(q)-\frac{q}{2M}K_2^{sing}\neq A(q) \tag{37}$$

Surface terms are the origin of this violation. For the derivation of (36) one has assumed the vanishing of surface terms. If one replaces the plane wave solution for the state by a wave packet, the state and therefore the matrix elements decrease sufficiently fast at spacial infinity and there are no surface terms. A experimental state is always a more or less localized wave packet rather than an exact plane wave. Therefore in regular gauge there are no surface terms and

$$K_1^{reg}(q)-\frac{q}{2M}K_2^{reg}=A(q) \tag{38}$$

is valid for all q. In order to work in singular gauge we have to choose a space-time manifold $I\!R^4\backslash\{0\}$ to exclude the unphysical singularity. This small hole should not affect the physics at large distances. Therefore all coordinate space intagrals are integrals over the domain $I\!R^4\backslash B_\varepsilon(0)$. Partial integration can now lead to surface terms at zero. The surface term is non-zero in the case of G_μ^{sing} as can be seen from (35). This is the reason for the inequality (37). It is surprising that not the slowly decaying regular gauge field causes a surface term at infinity but the strong singularity at the instanton centers in singular gauge leads to surface terms and to a violation of (36).

The following conclusions should be drawn:

1. not to consider gauge dependent objects like $K_{1/2}(q)$ at all or
2. save the relation (36) by using regular gauge although this violates the philosophy of [26] or
3. modify relation (36) by including the surface terms and be careful when performing partial integrations.

In the following discussion we take position 2.

9 Discussion

Comparing the results for the form factors $\Delta\Sigma^{GI} = G_1^{GI}(0)$, $G_2^{GI}(0)$, $A(0)$, $K_1(0) = K_1^{reg}(0)$ and $K_2(0) = K_2^{reg}(0)$ summarized in the last row of table 4 we clearly see that they are in contradiction. It is not possible to determine the remaining form factors in a way that they are consistent with (7) and (17). The most obvious contradiction is $\Delta\Sigma^{GI} \neq A(0)$. An opposite sign of the anomaly would at least be theoretical consistent and would lead to the naive expectations. The only candidate for this violation of the axial ward identities is the neglection of the non-zeromodes. All other approximations respect the symmetries of QCD as discussed in [2].

Forte [25] has derived the relation $\Delta\Sigma + A(0) = 0$ in the instanton model in the case of one quark flavor in quenched approximation and density expansion. $\Delta\Sigma$ was identified with $\Delta\Sigma^c$ and $K_2(0)$ was assumed to be zero (although not explicitly stated). Therefore $A(0) = K_1(0)$ and from (15) one can arrive at the welcomed result $\Delta\Sigma^{GI} = 0$.

In section 8 I have shown that the anomaly contributes to K_2 and not to K_1. This is the first discrepancy. Further, in section 6 I have shown that $\Delta\Sigma$ has to be identified with $\Delta\Sigma^{GI}$. This is the second discrepancy. It may turn out that the inclusion of non-zeromodes removes the discrepancies in a way, that leads to a phenomenological welcomed small $\Delta\Sigma^{GI}$. In the one instanton approximation the inclusion is managable and has been performed by [22] for the meson correlators. The consistent extension to the instanton liquid and to the quark form factors was not yet managable.

These problems might be compared to calculations of the η' mass. A brute force method of calculating the axial singlet meson correlator and extracting $m_{\eta'}$ by a spectal fit is not successful too. More elaborate arguments, given in [?] allowed a successful determination of $m_{\eta'}$. The instanton model was only used as a motivation for a selfdual model of QCD. Maybe the same model is able to solve the proton spin problem rather than a brute force calculation.

It might also be possible that the spin problem can not be solved on the level of individual constituent quarks formed out of current quarks but is connected with a strong interaction in the axial singlet channel between different constituent quarks. This possibility in connection with instantons is discussed in [23].

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