Monotone Conditional Complexity Bounds on Future Prediction Errors

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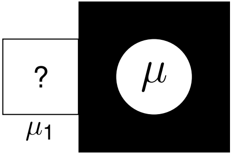
Outline

Sequence



finite alphabet $\mathcal{X} \ni x_1, x_2, \ldots$

μ is a measure

$$\mu_{i+1}(\cdot) = \mu(\cdot | x_1 \ldots x_i) = \frac{\mu(x_1 \ldots x_i \cdot)}{\mu(x_1 \ldots x_i)}$$

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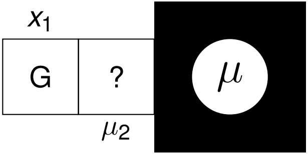

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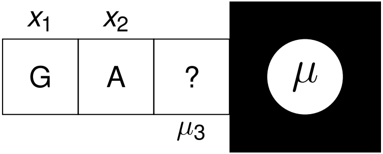

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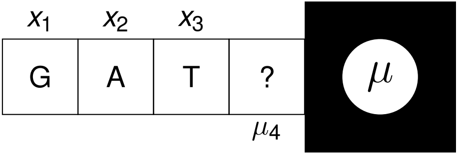

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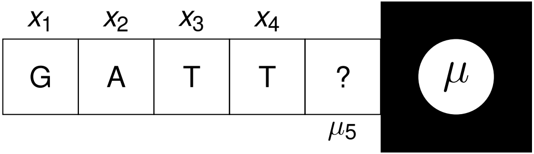

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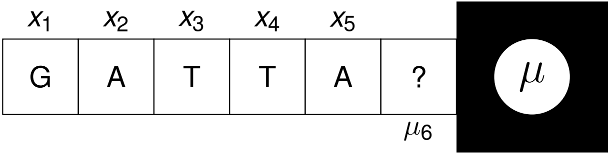

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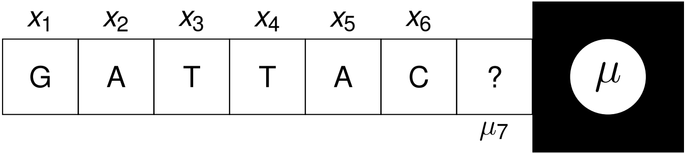

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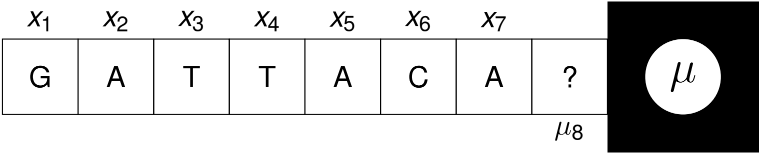

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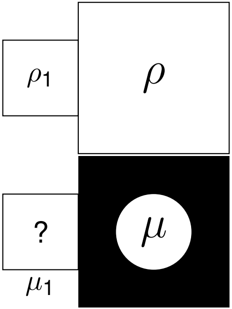
Predictor

Predictor ρ: $x_1, \ldots, x_i \quad \mapsto \quad \rho_{i+1}(\cdot) \approx \mu_{i+1}(\cdot)$



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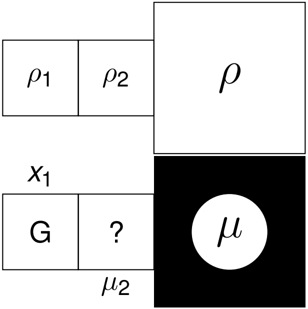

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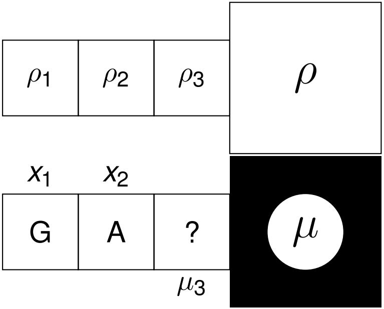

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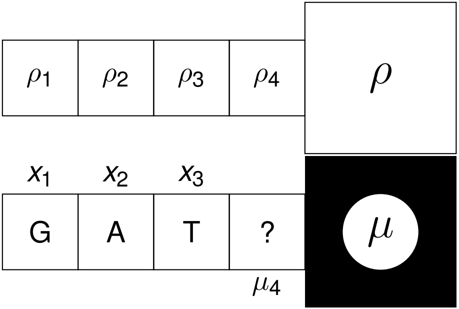

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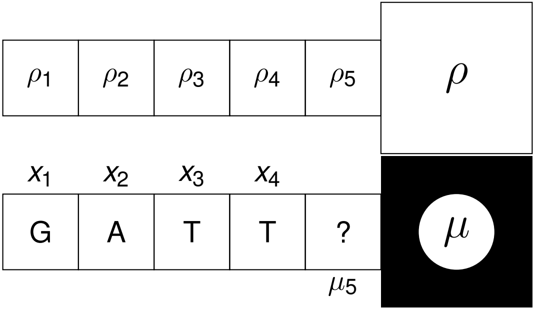

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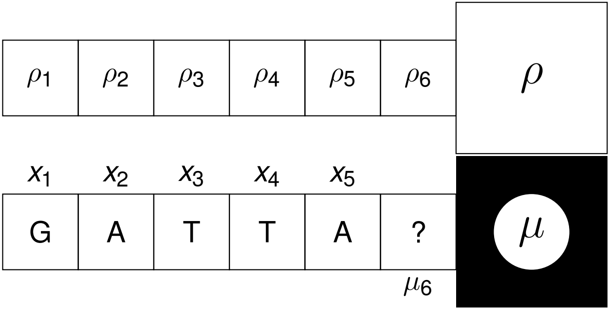

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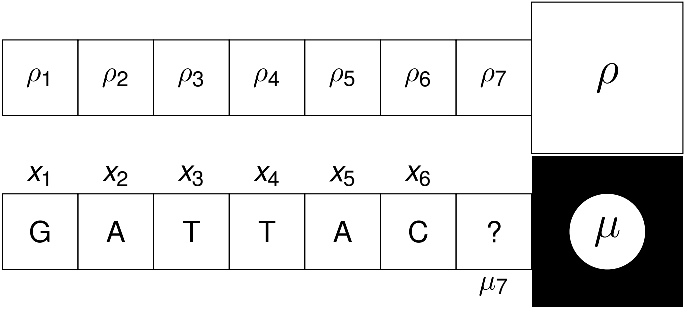

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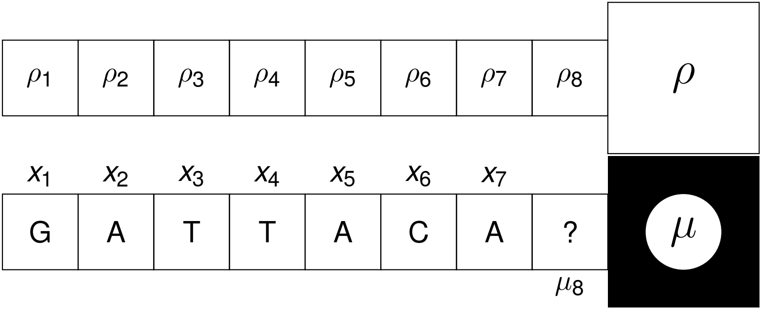


Quality of prediction

$\rho_i(\cdot) \approx \mu_i(\cdot)$ with high μ-probability:

$$\mathrm{Dist}(\rho, \mu) = \mathbf{E} \sum_{i=1}^{\infty} \mathrm{dist}_{x_1 \ldots x_i}(\rho_i, \mu_i) \quad = \sum_{x_1 x_2 \ldots} \mu(x_1 x_2 \ldots) \sum_{i=1}^{\infty} \mathrm{dist}_{x_1 \ldots x_i}(\rho_i, \mu_i)$$

$$\mathrm{dist}_{x_1 \ldots x_i}(\rho_i, \mu_i) = \frac{1}{\ln 2} \times$$

$$\sum_{a \in \mathcal{X}} (\rho_i(a) - \mu_i(a))^2 \quad \text{or} \quad \frac{1}{2} \left(\sum_{a \in \mathcal{X}} |\rho_i(a) - \mu_i(a)| \right)^2 \quad \text{or}$$

$$\sum_{a \in \mathcal{X}} \left(\sqrt{\rho_i(a)} - \sqrt{\mu_i(a)} \right)^2 \quad \text{or} \quad \sum_{a \in \mathcal{X}} \mu_i(a) \ln \frac{\mu_i(a)}{\rho_i(a)}$$

$$0 \leq \mathrm{Dist}(\rho, \mu) \leq D_\rho := \mathbf{E} \log_2 \frac{\mu(x_1 x_2 \ldots)}{\rho(x_1 x_2 \ldots)}$$

Intuitively: (for a deterministic μ)

$D_\rho \sim$ number of prediction errors

Solomonoff prior

$$\rho_i(\cdot) = \frac{M(x_1 \ldots x_i \cdot)}{M(x_1 \ldots x_i)} \qquad M(x) = \sum_{\mu} w_\mu \mu(x)$$

M is a Bayes mixture of all semi-computable semi-measures.

Theorem (Solomonoff 1964, 1978)

For any computable measure μ

$$\mathrm{Dist}(M, \mu) \leq D_M \overset{+}{\leq} K(\mu)$$

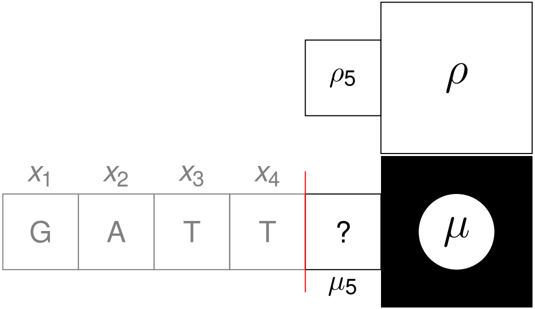
$K(\mu)$ is Kolmogorov complexity of μ

$\sim$ quantity of information in μ

$\sim$ the size of the shortest description of μ

Outline

Prediction with a priori information



$x_1, \ldots, x_n$ are fixed

$$\mathrm{Dist}(\rho, \mu | x_1 \ldots x_n) = \underset{x_{n+1} x_{n+2} \ldots}{\mathbf{E}} \sum_{i=n+1}^{\infty} \mathrm{dist}_{x_1 \ldots x_i}(\rho_i, \mu_i)$$

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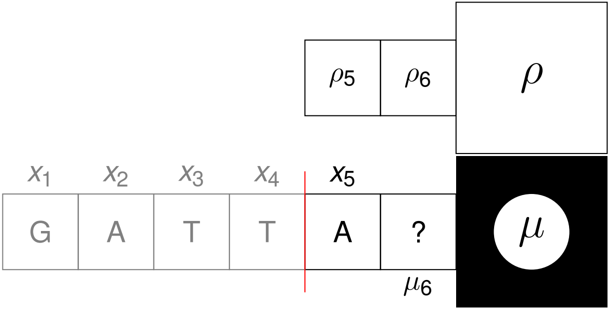

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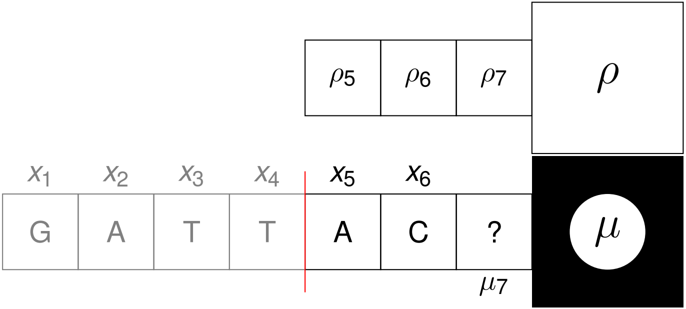

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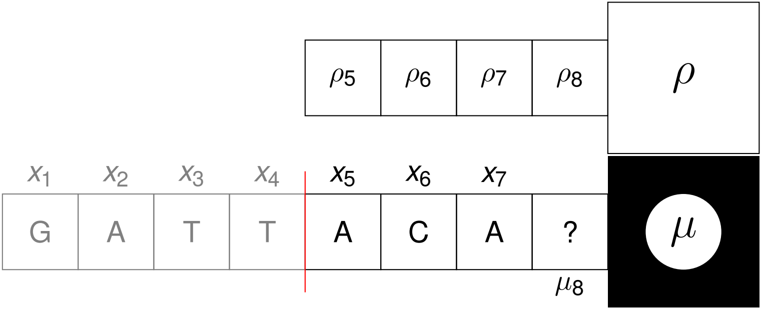

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The problem

$$x = x_1 \dots x_n \qquad \mathrm{Dist}(M, \mu | x) \leq D_M(x) := \mathbf{E}_y \log_2 \frac{\mu(y_1 y_2 \dots | x)}{M(y_1 y_2 \dots | x)}$$

For any computable measure μ, for any word x

$$\frac{\mu(y|x)}{M(y|x)} \leq \quad ?$$

We know

$$\log_2 \frac{\mu(y)}{M(y)} \stackrel{+}{\leq} K(\mu)$$

We want

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If x contains a lot of information about μ ($K(\mu|x)$ is small), prediction is easy.

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Prefix Kolmogorov complexity

Definition

$x, y \in \mathcal{X}^*$, U is a universal prefix machine, $p \in \{0,1\}^*$

$$K(y|x) = \min\{\ell(p) | U(p*, x) = y\}$$

Prefix machine

U gets finite x and infinite sequence α,
reads a finite part p of α, and halts with output y

Universal machine

for any other machine V: there is constant C

$$V(q,x) = y \quad \Rightarrow \quad \ell(q) \leq \ell(p) + C, \qquad \text{where } U(p,x) = y$$

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$K(\mu|x)$ bound

Theorem

For any computable measure μ and any $x, y \in \mathcal{X}^*$

$$\log_2 \frac{\mu(y|x)}{M(y|x)} \overset{+}{\leq} K(\mu|x) + K(\ell(x))$$

Corollary

1. $\mathrm{Dist}(M, \mu|x_1 \dots x_n) \overset{+}{\leq} K(\mu|x_1 \dots x_n) + K(n)$
2. $\mathrm{Dist}(M, \mu) \overset{+}{\leq} \min_n \{\mathbf{E}_{\ell(x)=n} K(\mu|x) + K(n) + \frac{2}{\ln 2} n\}$

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Example

$$\mathrm{Dist}(M,\mu) \overset{+}{\leq} \min_n \{ \mathbf{E}_{\ell(x)=n} K(\mu|x) + K(n) + \frac{2}{\ln 2} n \}$$

x_1

?

"number of errors" $\sim \mathrm{Dist}(M,\mu)$

Solomonoff bound:

$$\mathrm{Dist}(M,\mu) \lesssim K(\mu) \sim \text{"size of the image"} \approx 10^5$$

New bound:

$$\mathrm{Dist}(M,\mu) \lesssim K(\mu|x_1) + K(1) + \frac{2}{\ln 2} \sim \text{"small constant"}$$

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Outline

Structure of the bounds

If x is not μ-typical ($\mu(x) \approx 0$), then x is disinformation

prediction errors	$\Leftarrow$	information	$+$	quantity of disinformation
$\mathrm{Dist}(M, \mu)$	$\stackrel{+}{\leq}$	$K(\mu)$		
$\mathrm{Dist}(M, \mu\|x)$	$\stackrel{+}{\leq}$	$K(\mu\|x)$	$+$	$K(\ell(x))$
$\mathrm{Dist}(M, \mu\|x)$	$\stackrel{+}{\leq}$	$K(\mu)$	$+$	$K(d_\mu(x))$
$\mathrm{Dist}(M, \mu\|x)$	$\stackrel{+}{\leq}$	$K_*(\mu\|x*)$	$+$	$K(d_\mu(x))$

Randomness deficiency: $d_\mu(x) = \log_2 \frac{M(x)}{\mu(x)}$

d_μ is a measure of non-typicalness, $d_\mu(x)$ is small for most x
$d_\mu(x) = \ell(x) - K(x)$ for uniform μ

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$\mathrm{Dist}(M, \mu \vert x)$	$\overset{+}{\leq}$	$K(\mu \vert x)$	$+$	$K(\ell(x))$
$\mathrm{Dist}(M, \mu \vert x)$	$\overset{+}{\leq}$	$K(\mu)$	$+$	$K(d_\mu(x))$
$\mathrm{Dist}(M, \mu \vert x)$	$\overset{+}{\leq}$	$K_*(\mu \vert x*)$	$+$	$K(d_\mu(x))$

Randomness deficiency: $$d_\mu(x) = \log_2 \frac{M(x)}{\mu(x)}$$

d_μ is a measure of non-typicalness, $d_\mu(x)$ is small for most x
$d_\mu(x) = \ell(x) - K(x)$ for uniform μ

Structure of the bounds

If x is not μ-typical ($\mu(x) \approx 0$), then x is disinformation

prediction errors	$\Leftarrow$	information	$+$	quantity of disinformation
$\mathrm{Dist}(M, \mu)$	$\stackrel{+}{\leq}$	$K(\mu)$		
$\mathrm{Dist}(M, \mu\|x)$	$\stackrel{+}{\leq}$	$K(\mu\|x)$	$+$	$K(\ell(x))$
$\mathrm{Dist}(M, \mu\|x)$	$\stackrel{+}{\leq}$	$K(\mu)$	$+$	$K(d_\mu(x))$
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New conditional complexity

Definition (Prefix complexity monotone in conditions)

$x, y \in \mathcal{X}^*$, U is a universal twice-prefix machine, $p \in \{0,1\}^*$

$$K_*(y|x*) = \min\{\ell(p) | U(p*, x*) = y\}$$

Recall: conditional prefix complexity

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$$K_*(y|xz*) \leq K_*(y|x*)$$

$K_*(\mu|x*)$ bound

Theorem

For any computable measure μ and any $x, y \in \mathcal{X}^*$

$$\log_2 \frac{\mu(y|x)}{M(y|x)} \overset{+}{\leq} K_*(\mu|x*) + K(\lceil d_\mu(x) \rceil)$$

Corollary

$$\mathrm{Dist}(M, \mu|x_1 \dots x_n) \overset{+}{\leq} \min_{i \leq n}\{K(\mu|x_1 \dots x_i) + K(i) + K(d_\mu(x_1 \dots x_i))\}$$

For μ-typical x, $\mathrm{Dist}(M, \mu|x) \leq K(\mu|x') + O(\log \ell(x'))$

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Conclusion and Open Problems

We extended the Solomonoff results on online sequence prediction to the case when some initial part of the sequence is given.

- Informative initial segment reduces the future loss; this gives us improved total loss bounds if the alphabet is large
- The future loss $\Leftarrow$ information $+$ quantity of disinformation

Directions for further research:

- Future loss bounds for general Bayes mixtures
- Online classification instead of sequence prediction
- Technical properties of the new complexity

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